Analysis of Bifurcational Properties of the Mechanism RED

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Abstract - **In this paper is offered the method of analysis of bifurcational properties of dynamic systems represented by differential equations on the example of model of TCP-session of data communication at the management by a length of queue with the use of mechanism of congestion avoidance the RED.**

Keywords – **Stability, Bifurcation, Telecommunication, TCP-session, RED**

I. INTRODUCTION

With spreading set of the services communication, different network protocols, growing amount abonents and increase of their requirements to quality of service burning issue becomes the analysis and provided observability, controllability and, first of all, stability of the operation to network. However heuristic schemes and algorithms, prescribed in base existing facilities of the congestion avoidance quite often themselves provoke loss to stability of the operation TCN (Telecommunication Network). For example overloading of the “shortest” paths under realization mainly singlepath strategy routing (RIP, IGRP/EIGRP, OSPF, PNNI), or not always motivated a restriction of the length queue (RED, WRED) load flows to TCN (Traffic Shaping, Committed Access Rate) because of inadequate reaction on unstable nature to intensities of the network traffic [1]. So actual problem is a development new and/or improvement existing models and methods of the congestion avoidance, which have allowed to define area of the stable operation TCN in condition of the stochastic change the features of the subscription traffic, separate structured and functional network parameters.

II. ANALYSIS OF STABILITY AND BIFURCATION TCP-SESSION DATA TRANSMISSION

Under stability is understood nature of reactions of the dynamic system, which is TCN, on small disturbance of its condition. We shall take, that TCN will be stable, if small change of structural and functional parameter of network do not cause essential change of its condition. To present-day moment for analysis of the stability exists a lot of the methods, offered known scientist (Newton, Hamilton, A.M. Lyapunov, A.A. Andronov etc.).

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Broad spreading has got theory of bifurcation of the dynamic systems and theory of the catastrophes [2], by means of which possible analyzed causes and effects of the sudden uneven changes to behaviour of the dynamic system under small change its internal parameter or external conditions.

To estimate the area of stable or unstable TCN operation most exactly possible only at presence of the appropriate mathematical description – models of network process of the information exchange or management, taking into account stochastic nature and the dynamics of the network condition change. Thereupon deserve attention model that describe process of the change send rate data of TCP-session at the management by a length of queue with the use of mechanism RED (Random Early Detection) [3]. The model is based on consideration of the simplest IP-network fragment that consists of two network nodes, which use mechanism RED. Change of density of the flow (rate of data send) possible present in the manner of stochastic differential equation [3]:

$$\frac{d\lambda(t)}{dt} = \frac{1 - P_L(\lambda, t)}{R^2} - \frac{P_L(\lambda, t)}{2}\lambda^2(t), \qquad (1)$$

where $\lambda(t)$ –TCP-flow density; $P_L(\lambda, t)$ – probability losses of the segment; R – the channel response time.

Thereby, mathematical model of session data transmission at the management by a length of queue with the use of mechanism RED (1) carries clearly defined nonlinear nature. Defining and analyzing the area of the stable network functioning, based on presented model, is allow opportunely reveal and prevent the TCN overloads and losses of packages (the segment) related to it.

In that event if dynamic system introduces by differential equation, for analysis of stability mode its operation applicable theory of bifurcation and is realized method, founded on analysis bifurcation characteristic of the investigated system [2]. We shall analyse stability of the conditions of the dynamic system, given by equation (1) for event of independence of the loss probability of the segment data P_L from time and density of the flow $\lambda(t)$, i.e. $P_L(t, \lambda) = \text{const}$.

Step 1. According to method on the first step searching of the stationary states of the system (1) is realized. For this states it is typical $\frac{d\lambda(t)}{dt} = 0$. As a result of solution of the equation we get two root $\lambda_1^0 = \sqrt{\frac{2(1 - P_L)}{P_L R^2}}$ and $\lambda_2^0 = \sqrt{\frac{2(1 - P_L)}{P_L R^2}}$.

Step 2. On the second step it is produced analysis of stability about got stationary point by searching the dynamics of behaviors (the evolutions) of the small disturbance, inserted into initial equation. Let $\lambda^0(t)$ there is certain partial solution of the equation (1). We research stability of this solution (state), for what shall insert in consideration variable $y(t)$, which will assign small deviation from partial solution:

$$y(t) = \lambda(t) - \lambda^0(t), \tag{2}$$

where $\lambda(t)$ – perturbed solution.

Marked left part of equation (1) through F, evolution of the small indignation $y(t)$ is possible to present in the manner of linear equation

$$\dot{y} = A(t)y, \text{ where } A(t) = \left.\frac{dF}{d\lambda}\right|_{\lambda=\lambda^0(t)}, \tag{3}$$

which is get subject to decompositions of the function F in power series in vicinities of partial solution $\lambda^0(t)$ [4]:

$$F(y) = \left.\frac{dF}{dx}\right|_{x=x^0(t)} y(t) + \left.\frac{d^2F}{dx^2}\right|_{x=x^0(t)} y^2(t) + \ldots \tag{4}$$

We shall consider equation for disturbances (3) with reference to first stationary state λ_1^0:

$$\dot{y} = (P_L\lambda_1^0)y = (\frac{\sqrt{2P_L(1-P_L)}}{R})y = Ay, \tag{5}$$

where $A = \left.\frac{dF}{d\lambda}\right|_{\lambda_1^0} = \frac{\sqrt{2P_L(1-P_L)}}{R}$.

Solution of the equation (4) will $y = \exp(At)$. Disturbance exponential decays at time (A there is negative number). This means that state λ_1^0 is stably. Since second state λ_2^0 differs from first only by sign, solution of the equation (5) in this case will be exponential increase at time (fig. 1). Stationary state λ_2^0 is unstable.

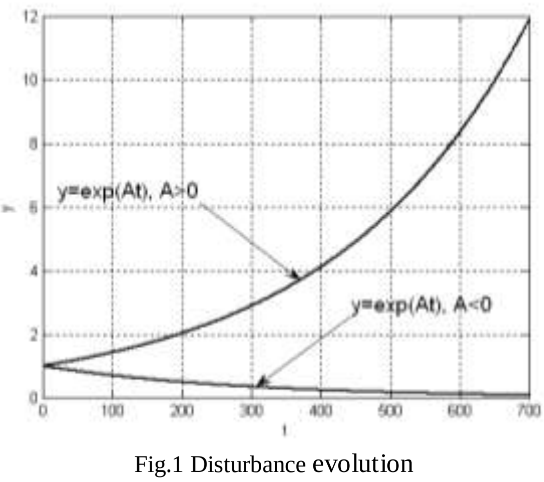

Fig.1 Disturbance evolution

Considering, that $\lambda(t)$ defines density of the flow segment data, i.e. $\lambda(t) > 0$, that solution of the equation (1) can be only positive root. Consequently, stationary state λ_1^0 unstable that indicate of stability of the model (1) within the framework of the offered method.

Step 3. Next step is a mathematical analysis of bifurcation, that in the wide sense identify various qualitative reorganizations or metamorphoses in behaviour of the system under changes of parameters, from which its work depends [2]. In this instance the stability is defined by sign of the derivative in right part of equation (1) in stationary point, i.e. sign of the value A (3). At a reduction of the value of first item $a = \frac{1-P_L(t)}{R^2}$ in the equation (1) magnitude A is approximate to zero and finally under $a = 0$ parameter A is turn to zero. Then, if $a < 0$ there are stationary states is not present. In this case $\lambda_{1,2}^0 = \pm j\sqrt{\frac{2(1-P_L)}{P_L R^2}}$ (j – imaginary unit), i.e. when $a < 0$ become purely imaginary. Point of bifurcations will be importance $A = 0$ and such event appears when $P_L = 1$. Thereby, until the buffer on network node is completely loaded, since certain moment of time with increase the volume of incoming traffic increases probability of the segment loss. At the same time the equation (1) has one stationary solution, which is stable. However, when buffer is overfilled, all coming on node packages are rejected with probability $P_L = 1$, that corresponds to occurrence the phenomena of bifurcations. All aforesaid signify of adequacy of the investigated model to real process of data communication.

III. Conclusion

Approach to analysis of stability of the operation TCN on basis of use the bifurcation theory is offered. The features of the using the offered approach are demonstrated on example of the searching TCP-session of data communication at the management by a length of queue with the use of mechanism RED. The process of changing of data rate is presented by nonlinear differential model. Results of the analysis indicate that process of the congestion avoidance possesses stability as a whole at the condition $P_L(t,\lambda) = const$. Point of bifurcations corresponds to event, when are rejected all incoming segments, i.e. $P_L = 1$. It is planned to develop of the more complex models of the losses, when $P_L(t,\lambda) \neq const$ for the purpose of discovery and possible narrowing the area of stability of the solutions of used model.

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