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RESEARCH ON TENSOR MODEL OF MULTIPATH ROUTING IN TELECOMMUNICATION NETWORK WITH SUPPORT OF SERVICE QUALITY BY GREATE NUMBER OF INDICES

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A tensor data-flow model of multipath routing with guarantee of quality by several indices is presented. The novelty of the model consists in transfer to the updated version of the flow storage conditions taking into account packets possible losses caused by overflow of the buffer queue on the network routers. Due to the tensor formalization of the network model it was possible to receive in the analytical form the conditions of imparting quality to simultaneous service by multiple various indices, namely, the transmission rate, mean delay and packages loss probability. The results of the research have shown the advantages of the offered method; they have made themselves evident in the improvements in the end-to end multipath delay, increase in the transmission rate, packets jitter minimization resulted from realization of the routing multipath strategy, increase in the probability of meeting the QoS-requirements and decrease in service abandonments caused by nonfulfillment of the specified quality level.

KEY WORDS*: traffic control, quality of service, tensor model, end-to-end multipath delay*

1. INTRODUCTION

Integral attributes of the modern telecommunication networks (TCN) are their multiservice, multimedia nature of the traffic being transmitted and availability of requirements on quality of service (QoS) simultaneously by multiple dissimilar indicators, namely, the transmission rate, mean delay, jitter and packets loss probability. Satisfaction of the complete totality of requirements represents sufficiently complicated theoretical and applied problem as the ultimate user service quality is a

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result of interaction of the multiple network devices, protocols and mechanisms functioning practically at all levels of the OSI (Open Systems Interconnection) model. The key to successful performance of guarantees as to the end-to-end quality of service (the end-to-end QoS) involves consistency and subordination of every separate network function to a common goal – QoS. Such a consistency can be attained in the framework of the ultimate users' transparent interaction provided, among other things, with the traffic control means at the network level, where the key positions are assigned to the routing problem [1,2].

The routing challenge under the influence of general trends of telecommunication networks development transforms into the rout search problem or the multitude of routes along which the volume of free resources makes it possible to meet the specified QoS requirements (QoS based routing). In this case none of the known protocol decisions meet the given statement merely because the shortest way search algorithms of the Bellman- Ford and Diykstra, forming the basis of them, do not make it possible to operate with multiple dissimilar QoS indices simultaneously [3,4]. Moreover, the graph models do not meet other no less important requirements, namely, the requirement of the balanced used of network resources, which is the basis of the Traffic Engineering concept aimed at the increase in the network functioning efficiency and the service quality as a whole [2]. Then the routing problem can be stated as the problem of searching the set of ways and balancing of the load between them, with which the required numerical values of the key QoS indices are guaranteed. Such a statement, first, assumes the traffic flows' manipulation, and therefore it defines the only possible class of models for its solution - the traffic control data-flow models, and, second, it requires introduction of the quality service indices in the explicit form into the model.

2. ANALYSIS OF KNOWN DECISIONS IN THE FIELD OF MATHEMATICAL SIMULATION OF ROUTING PROBLEMS IN TELECOMMUNICATION NETWORK

It is important to note that at present a sufficiently large number of the routing data-flow models are known, which, depending on the completeness of consideration of structural-functional TCN design, are presented with linear [5,6], nonlinear algebraic [7,8], Diophantine [9], difference-differential [10-12] and tensor [13,14] equations of state of the TCN. A great deal of publications is devoted to different problems of survey and comparative analysis of different routing models and methods [15-18]. According to the results of analysis it was established that the general tendency at using such models and methods consists in a significant widening of possibilities to analyze the routing processes in the TCN based on more complete inclusion of singularities of the service quality and the load balancing provision processes.

In this case some complication of the routing mathematical models, caused by realization of the nonstandard theoretical approaches and resulting in the increase in computational complexity of the finite protocol solutions, is justified as a whole by application of new technological paradigms of the prospective TCN construction. The

SDN (Software Define Networking) ideas emergence and active introduction should be assigned to the related concepts [19]. It is assumed in the frameworks of the SDN, that the tasks laid on the modern routers are simplified noticeable as a part of them is shifted off to the network operating system developed on the special route servers. As the efficiency of such servers exceeds noticeably the possibilities of the network routers the problem, as related to minimization of route solutions computational complexity, has assumed a secondary importance giving way to the requirements of rising the general level of consistency and coherence of decisions of the problems complex in the traffic control, service quality provision.

As to concerning the flow-oriented models development in the direction of the service quality provision and load balancing the tensor approach, which has proved itself as an effective means for integral and multidirectional description of the TCN, deserves special attention. The tensor models are built on the basis of simultaneous and complementary use of information on the structural and functional design of the TCN, this results, on the one hand, in the increase in the mathematical description adequacy and, on the other hand, it complicates formalization of the traffic control processes in the network and, respectively, calculation of control decisions with their aid. It is important to understand that complication on the stage of mathematical description should "pay for itself" with higher (as compared to the available analogues) efficiency of the route solutions and service quality being achieved. In this connection the aim of this paper consists, first, in description of particularities of tensor modeling of the traffic control problems with a service quality provision using the multipath routing as an example, second, demonstration of the offered approach advantages through estimation of the final decisions efficiency by the key QoS indicators.

3. ROUTING FLOW-ORIENTED MODEL TAKING INTO ACCOUNT PACKAGES LOSSES IN TELECOMMUNICATION NETWORKS

In the frameworks of the routing model being developed the TCN structure is described with a one-dimensional network $S=(U,V)$, where $U=\left\{u_i, i=\overline{1,m}\right\}$ is a set of zero-dimensional simplexes – nodes (routers) of a network, and $V=\left\{v_z=(i,j); z=\overline{1,n}; i,j=\overline{1,m}; i\neq j\right\}$ is a set of one-dimensional simplexes – branches of a network, where the branch $v_z=(i,j)$ simulates the z-th communication channel (CC) which unites the i-th and j-th TCN routers. Thus, the characteristics and parameters relating to the network communication channels will be denoted in this work by a single or double index depending on the consideration aspect of the processes taking place in the TCN. In the first case, when the channel is considered as an independent object, the sequential numbering of branches (channels) will be used, and in the second case, when accounting of the CC position in the network is important, the numbering will take place according to the nods numbers. For example, for each of the CC being simulated with the $v_z = (i, j) \in V$ branch, the traffic capacity measured in packages per second (1/s) will be symbolized both by φ_z and by $\varphi_{(i,j)}$.

Each routing node of the network has several interfaces through which it transmits packets to the nods-neighbors incident to it. At that the interfaces numbers for each node taken separately correspond to the numbers of the neighbor nods connected through them. The routing problem solution consists in calculation of a set of the route variables $x^k_{(i,j)}$, each of them characterizes intensity part of the k -th flow directed from the i -th node to the j -th node through the corresponding, i.e., j -th interface. In the process of the TCN functioning the packets losses can emerge in the nods interfaces (routers) caused by the queues overload created in these nodes. Let us denote the packets losses probability by $p_{(i,j)}$ on the j -th interface of the i -th node by reason of the overload. Then the expression $x^k_{(i,j)}\left(1 - p_{(i,j)}\right)$ (Fig. 1) characterizes the intensity part of the k -th flow in the channel $(i, j) \in V$ and the product $x^k_{(i,j)} p_{(i,j)}$ defines numerically the intensity part of the k -th flow which has received a denial of service on the j -th interface of the i -th node.

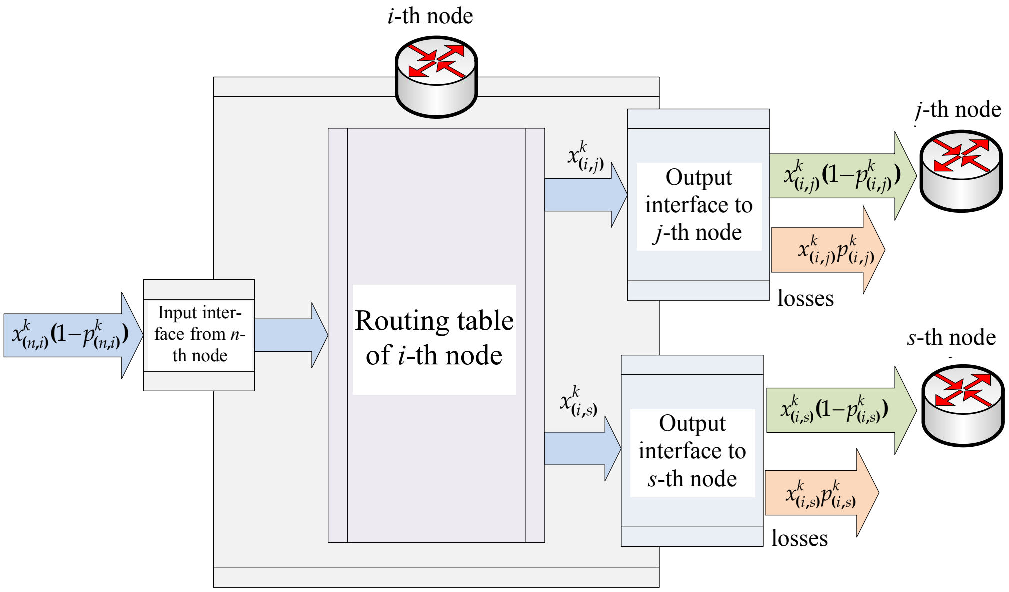


FIG. 1: Simplified architecture of the i -th node of the network at routing of the k -th flow

In the frameworks of the offered routing model to account for the possible packets losses on the TCN nodes the condition of the flow conservation assumes the form:

$$\begin{cases} \sum\limits_{j:(i,j)\in V} x_{(i,j)}^k = 1 \text{ at } k\in K,\ i = s_k; \\ \sum\limits_{j:(i,j)\in V} x_{(i,j)}^k - \sum\limits_{j:(j,i)\in V} x_{(j,i)}^k (1-p_{(j,i)}) = 0 \text{ at } k\in \mathrm{K},\ i \ne s_k, d_k; \\ \sum\limits_{j:(j,i)\in V} x_{(j,i)}^k (1-p_{(i,j)}) = \varepsilon^k \text{ at } k\in K,\ i = d_k, \end{cases} \quad (1)$$

where K is a set of flows in the network; s_k is a node-sender and d_k is a node-receiver for the packets of the k-th flow; ε^k is a part of the k-th flow served by the network, i.e., the packets of which were successfully delivered to the node-recipient.

In the course of calculations the interface, functioning according to the Tail Drop scheme, can be presented, for example, in the form of the mass service with denials of the $M/M/1/N$ type [6], in the frameworks of this system the packets losses probability can be calculated as

$$p = p\left(q < \Theta_b\right) = \frac{(1-\rho)(\rho)^N}{1-(\rho)^{N+1}}, \quad (2)$$

where $\rho = \dfrac{\lambda}{\varphi}$ is the channel load coefficient; q is the current queue average length; $N = \Theta_b + 1$ is the maximal number of packets which can be contained in the interface including the buffer (Θ_b) and the channel itself; λ is the packet intensity of the combined flow in the considered channel, 1/s. For clarity the indices, characterizing affiliation with one or another node (router), and the flow number are omitted in the expression (2). Taking into account the possible packets losses, the flow intensity in the channel $(i,j)\in V$ is calculated in the frameworks of the above symbols as

$$\lambda_{(i,j)} = \sum_{k\in K} \lambda_k^{\langle r\rangle} x_{(i,j)}^k \left(1 - p_{(i,j)}\right), \quad (3)$$

where $\lambda_k^{\langle r\rangle}$ is an average intensity of the k-th flow entering in the network for service which sets QoS requirements to packets transmission speed.

It should be noted that the drop of the packets from the queue can take place not only due to its actual overflow, realizing the Tail Drop scheme, but preventively in accordance with mechanisms of active queue management (AQM) [1].

To realize the multipath strategy of routing with the load balancing, the restrictions of the following form are imposed on the control variables

$$0 \le x_{(i,j)}^{k} \le 1 . \tag{4}$$

To ensure controllability over the process of the channels and queues overload fight, i.e., to meet the $\rho < 1$ condition, the following limitations are introduced into the model structure:

$$\lambda_{(i,j)} < \varphi_{(i,j)} \;\; , \;\; (i,j) \in E . \tag{5}$$

Because of the random nature of the modern mainly multimedia traffic the fulfillment of the conditions (5) is only the necessary one but not sufficient condition of the packets losses absence on the TCN nodes due to a possible queues overflow.

4. TENSOR GEOMETRIZATION OF TELECOMMUNICATION NETWORK

Let us introduce the anisotropic space-structure formed with a set of closed and open paths (circuits and node pairs) for the TCN tensor description. Dimension of such a space, being a structural invariant, is defined with a general quantity of branches in the network and is equal to *n* [13,14,20]. In this connection a set of structures, fitting different versions of *n* branches connections, can be treated as a set of particular coordinate systems in the introduced *n*-dimensional space. Transformation of the network structure with preservation of the initial number of branches or transition from one set of independent routes to another can be interpreted as the coordinate system transformation. Thus, each independent route defines the coordinate axis in the frameworks of the considered space-structure.

Let the network S be a connected one, i.e., let it contain one connected component, then the cyclomatic number $\mu(S)$ and rank $\phi(S)$ of the network define in it, respectively, a number of the basic circuits and node pairs stipulating the following expressions validity:

$$\phi(S) = m - 1 , \quad \mu(S) = n - m + 1 , \quad n = \phi(S) + \mu(S) . \tag{6}$$

In the n - dimensional space the basic circuits and node pairs define the basis being in conformity with the network structure. By means of the basic routes it is possible to express any other route of the network, where the algebraic sum is the route through all summands of the sum in accordance with their orientation [13,14,20].

For the consequent formalization of the routing problems solution in the introduced n-dimensional space let us carry out the tensor description of the TCN using the mixed bivalent tensor (once a covariant and once contra variant)

$$Q = T \otimes \Lambda, \tag{7}$$

where $\otimes$ is the sign of the direct tensor multiplication. The components of the tensor Q represent univalent covariant tensor of T packets mean delays and univalent contra variant tensor of Λ flow packet intensity values. Then the expression (7) in the index form takes the following form

$$q_j^i = \tau_j \lambda^i, \quad i, j = \overline{1,n}, \tag{8}$$

where τ_j is the packet transmission delay in the j-th coordinate route (c); λ^i is the packet intensity of the flow transmitted by the i-th basis route (1/c).

The following two coordinate systems (CS) will be taken into account, when considering the tensor (7): the network branches coordinate system and the coordinate system of circuits and node pairs. In the frameworks of the given CS data, as it will be shown below, the required and known views of Q tensor different components will be specified, based on them it is possible to get a solution of the formulated problem. Variance of tensor (8) components is substantiated in [13,14,20], where it was shown that characteristics which, at transition from one CS to another, are transformed according to the law of conservation of the flow, for example, the traffic flow intensity are contra-variant values, and parameters, which are transformed in accordance with the additive law, for example, packets mean delay, are covariant values. In the frameworks of the chosen example, when the interface is simulated in the form of the system of queuing with failures of the $M/M/1/N$ type [6], a mean packets delay in the random communication channel of the TCN is approximated by the expression

$$\tau = \frac{\rho - \rho^{N+2} - (N+1)\rho^{N+1}(1-\rho)}{\lambda\left(1-\rho^{N+1}\right)(1-\rho)}. \tag{9}$$

For the next tensor generalization the expression (9) will be written in the following form

$$\tau_i^v = \frac{\rho_i^v - \left(\rho_i^v\right)^{N_i^v+2} - \left(N_i^v+1\right)\left(\rho_i^v\right)^{N_i^v+1}\left(1-\rho_i^v\right)}{\left(1-\left(\rho_i^v\right)^{N_i^v+1}\right)\left(1-\rho_i^v\right)\left(\lambda_i^v\right)^2}\lambda_v^i, \; i = \overline{1,n}, \tag{10}$$

where i is the number of the communication channel in the TCN, and the index v indicates that all parameters of the expression (10) are referred to the network branches.

According to the axiom of the second generalization of G. Crone [20] the system of equations (10) can be substituted for the following vector equation

$$\Lambda_v = G_v T_v , \tag{11}$$

where $\Lambda_v = \begin{bmatrix} \lambda_v^1 & \dots & \lambda_v^i & \dots & \lambda_v^n \end{bmatrix}^t$ and $T_v = \begin{bmatrix} \tau_1^v & \dots & \tau_i^v & \dots & \tau_n^v \end{bmatrix}^t$ is the projection of tensors Λ and T in the coordinate system of the network branches represented in the form of the vectors of the flows intensities and mean packets delays, respectively, in the branches of the network of the n dimension; $[\cdot]^t$ is the transposition operation; $G_v = \left\| g_v^{ij} \right\|$ is the diagonal matrix of the $n \times n$ dimension, the main diagonal elements of it are calculated according to the expressions (10) related to the corresponding branches of the network $\left\{ v_i, i = \overline{1,n} \right\}$, i.e.,

$$g_v^{ii} = \frac{\left(1-(\rho_i^v)^{N_i^v+1}\right)\left(1-\rho_i^v\right)\left(\lambda_i^v\right)^2}{\rho_i^v - \left(\rho_i^v\right)^{N_i^v+2} - \left(N_i^v+1\right)\left(\rho_i^v\right)^{N_i^v+1}\left(1-\rho_i^v\right)} . \tag{12}$$

Let us note that the $M/M/1/N$ model is not the only possible one for the analytical description of the net interfaces: with this aim in view the systems of queuing of another type can be used, and in the general case there can be functional equations derived in the frameworks of other mathematical apparatus. The choice of the net interfaces functioning model is defined by the requirements for the adequacy of the mathematical description and limitations associated with the computational complexity of the consequent calculations. For example, the above mentioned model $M/M/1/N$ describes adequately the SMTP official traffic flows, and to describe the IP, FTP, TCP, HTTP data flows, for the purpose of accounting their self-similar nature, more complicated, but making it possible to take into account long-term dependences, the mass service system $SS/M/1/N$ should be used, where the SS symbol points to the input flow self-similarity [21]. In the frameworks of the $SS/M/1/N$ model for the mean delay of the packets τ_i^v in the i-th channel the expression [21] is used:

$$\tau_i^v = \frac{q_i + \frac{\rho_i^v}{\chi_i} f(H)}{f(H) \cdot \sum_{k \in K} \lambda_i^v} + \frac{1}{\varphi_i^v} =$$

$$= \frac{\frac{1}{\chi_i \varphi_i^v}}{1 - \left[\frac{\rho_i^v}{\chi_i} f(H)\right]^{N_i^v + 2}} \cdot \frac{\left\{1 - \left(N_i^v + 1\right)\left[\frac{\rho_i^v}{\chi_i} f(H)\right]^{N_i^v} + N_i^v \left[\frac{\rho_i^v}{\chi_i} f(H)\right]^{N_i^v + 1}\right\}}{1 - \frac{\rho_i^v}{\chi_i} f(H)} + \frac{1}{\varphi_i^v}, \tag{13}$$

$$q_i = \frac{\frac{\rho_i^v}{\chi_i} f(H)}{1 - \left[\frac{\rho_i^v}{\chi_i} f(H)\right]^{N_i^v + 2}} \times$$

$$\times \frac{\left\{1 - \left(N_i^v + 1\right)\left[\frac{\rho_i^v}{\chi_i} f(H)\right]^{N_i^v} + N_i^v \left[\frac{\rho_i^v}{\chi_i} f(H)\right]^{N_i^v + 1}\right\}}{1 - \frac{\rho_i^v}{\chi_i} f(H)} - \frac{\rho_i^v}{\chi_i} f(H), \tag{14}$$

where q_i is a mean queue length on the i-th network interface, i is the number of the communication channel in the TCN, $f(H)$ is the function which takes into account a self-similar nature of the traffic arriving to the TCN; χ_i is the probability of absence of retransmission packets on the i-th network interface which arises, for example, when the packet is transmitted according to the protocol TCP/RED (Transmission Control Protocol/Random Early Detection).

Then the elements of the principle diagonal of the matrix G_v will be calculated as

$$g_v^{ii} = \lambda_v^i \times$$

$$\times \left[\frac{\frac{1}{\chi_i \varphi_i^v}}{1 - \left[\frac{\rho_i^v}{\chi_i} f(H)\right]^{N_i^v + 2}} \cdot \frac{\left\{1 - \left(N_i^v + 1\right)\left[\frac{\rho_i^v}{\chi_i} f(H)\right]^{N_i^v} + N_i^v \left[\frac{\rho_i^v}{\chi_i} f(H)\right]^{N_i^v + 1}\right\}}{1 - \frac{\rho_i^v}{\chi_i} f(H)} + \frac{1}{\varphi_i^v}\right]^{-1}. \tag{15}$$

There are unambiguous conversion rules for the coordinates of any geometrical objects in going from one basis to another one as the considered systems of coordinates of branches, circuits and node pairs of the network are introduced for the same n-dimensional space. The coordinate transformation rules have the linear nature for the tensors according to the definition [20] and they are formalized with nonsingular squared matrix of $n \times n$ dimension:

$$\Lambda_v = C\,\Lambda_{\pi\eta}, \tag{16}$$

where $\Lambda_{\pi\eta}$ is the projection of the tensor Λ in the system of coordinates of circuits and node pairs presented in the form of the vector of n dimension, and C is the matrix of the contra variant transformation.

The projections of tensors of the flows intensities Λ and mean delays T, represented by the vectors $\Lambda_{\pi\eta}$ and $T_{\pi\eta}$, respectively, have the dimension n and the following constituents in the system of coordinates of circuits and node pairs

$$\Lambda_{\pi\eta} = \begin{bmatrix} \Lambda_\pi \\ -- \\ \Lambda_\eta \end{bmatrix};\ \Lambda_\pi = \begin{bmatrix} \lambda_\pi^1 \\ \vdots \\ \lambda_\pi^j \\ \vdots \\ \lambda_\pi^\mu \end{bmatrix};\ \Lambda_\eta = \begin{bmatrix} \lambda_\eta^1 \\ \vdots \\ \lambda_\eta^p \\ \vdots \\ \lambda_\eta^\phi \end{bmatrix},\ T_{\pi\eta} = \begin{bmatrix} T_\pi \\ -- \\ T_\eta \end{bmatrix};\ T_\pi = \begin{bmatrix} \tau_1^\pi \\ \vdots \\ \tau_j^\pi \\ \vdots \\ \tau_\mu^\pi \end{bmatrix};\ T_\eta = \begin{bmatrix} \tau_1^\eta \\ \vdots \\ \tau_p^\eta \\ \vdots \\ \tau_\phi^\eta \end{bmatrix}, \tag{17}$$

where Λ_π, T_π are the flows intensities vectors taking place in the network circuits and delays in these circuits, respectively; Λ_η, T_η are the vectors of intensity of the flow arriving to its nodes or guided through them to the network customers and subjected to delays in the process; λ_π^j is the flow intensity in the circuit π_j of the net; λ_η^p is the intensity of the external flow incoming into the network and descending from the network through the node pair η_p; τ_j^π, τ_p^η are the mean delays of the packages in the circuit π_j and between the pair of nodes η_p of the network, respectively.

The covariant nature of the delays tensor T stipulates the following coordinate transformation law:

$$\grave{O}_v = A\ \grave{O}_{\pi\eta}, \tag{18}$$

where A is the matrix of the covariant transformation of $n \times n$ dimension connected with the matrix C by the orthogonality condition $\tilde{N}A^t = I$; I is the identity matrix of $n \times n$ dimension.

The expression (11), derived for the CS of the branches, preserves its form invariable both in the CS of the circuits and node pairs according to the postulate of G. Crone's second generalization:

$$\Lambda_{\pi\eta} = G_{\pi\eta}\ T_{\pi\eta}\ . \tag{19}$$

Then according to the inverse tensor sign the G tensor represents double contravariant metric tensor, its projections are transformed in the following way at variation of the coordinate system of its inspection:

$$G_{\pi\eta} = A^t G_v\ A\ , \tag{20}$$

where $G_{\pi\eta}$ is the projection of the G tensor in the system of coordinates of circuits and node pairs.

4. FORMALIZATION OF CONDITIONS OF SERVICE QUALITY ASSURANCE SIMULTANEOUSLY BY A SET OF VARIOUS INDICES

Let us consider the problem of derivation of the quality assurance conditions of the TCN service in the frameworks of the tensor model (6)-(20). The following data act as the initial ones:

1) the initial structure of the TCN, this makes it possible to form the coordinate transformation matrices A and C;

2) throughputs of communication channels and the sizes of the buffer capacity in the network nodes, models of the traffic itself and the process of its service which define the contents of metric tensor projection matrices in the CS of branches (12) or (15);

3) direction of the flow transmission with indication of the input and output in the network;

4) numerical values of the main service quality indices: the required flow intensity $\lambda^{\langle r\rangle}$, the admissible end-to-end mean delay $\tau_{\langle add\rangle}$ and probability of packets losses $p_{\langle add\rangle}$. Let us agree that the first index is assigned to the transmitter-receiver pair at numbering of the network node pairs.

Let us write the expression (19) with regard to (17) in the following form

$$\left\| \begin{array}{c} \Lambda_\pi \\ --- \\ \Lambda_\eta \end{array} \right\| = \left\| \begin{array}{ccc} G^{\langle 1\rangle}_{\pi\eta} & | & G^{\langle 2\rangle}_{\pi\eta} \\ --- & + & --- \\ G^{\langle 3\rangle}_{\pi\eta} & | & G^{\langle 4\rangle}_{\pi\eta} \end{array} \right\| \cdot \left\| \begin{array}{c} T_\pi \\ --- \\ T_\eta \end{array} \right\|, \tag{21}$$

where $\begin{Vmatrix} G^{\langle 1\rangle}_{\pi\eta} & | & G^{\langle 2\rangle}_{\pi\eta} \\ --- & + & --- \\ G^{\langle 3\rangle}_{\pi\eta} & | & G^{\langle 4\rangle}_{\pi\eta} \end{Vmatrix} = G_{\pi\eta}$, in this case $G^{\langle 1\rangle}_{\pi\eta}$, $G^{\langle 4\rangle}_{\pi\eta}$ are the square sub matrices of $\mu\times\mu$ and $\phi\times\phi$ dimensions, respectively, $G^{\langle 2\rangle}_{\pi\eta}$ is the sub matrix of $\mu\times\phi$ dimension, $G^{\langle 3\rangle}_{\pi\eta}$ is the sub matrix of $\phi\times\mu$ dimension.

Based on the physical features of T_π components, the fulfillment of the following condition is required

$$T_\pi = 0, \tag{22}$$

this guarantees the absence of loops in the routs and equal mean packets delay along each of them.

Then, according to (22) from the expression (21), the following expression takes place

$$\Lambda_\eta = G^{\langle 4\rangle}_{\pi\eta}\, T_\eta . \tag{23}$$

Let us present vectors Λ_η and T_η from (23) in the following form

$$\Lambda_\eta = \begin{Vmatrix} \lambda^1_{(\eta)} \\ --- \\ \Lambda_{\eta-1} \end{Vmatrix}, \quad \Lambda_{\eta-1} = \begin{bmatrix} \lambda^2_\eta \\ \vdots \\ \lambda^p_\eta \\ \vdots \\ \lambda^\phi_\eta \end{bmatrix}, \quad T_\eta = \begin{Vmatrix} \tau^{(\eta)}_1 \\ --- \\ T_{\eta-1} \end{Vmatrix}, \quad T_{\eta-1} = \begin{bmatrix} \tau^\eta_2 \\ \vdots \\ \tau^\eta_p \\ \vdots \\ \tau^\eta_\phi \end{bmatrix}. \tag{24}$$

In this case coordinates of the vector Λ_η point toward the intensity of the flow between the reference node (as a rule, it is the sender node) of the network and other nodes of the given network. For the transient nodes, not being the end addressees, the sub-vector $\Lambda_{\eta-1}$ (24) coordinates, i.e., the λ^j_η value, should define the packets losses intensity the total one for each j-th node by itself over all its interfaces. For the node-receiver one of the coordinates (the first one, as a rule) of the vector Λ_η defines the intensity of the flow served by the network

$$\lambda^1_{(\eta)} = \lambda^{\langle r\rangle} - \lambda^{\langle l\rangle}, \tag{25}$$

$$\lambda^{\langle l\rangle}=\sum_{j=2}^{\phi}\lambda_{\eta}^{j} \tag{26}$$

where $\lambda^{\langle l\rangle}$ is the total intensity of the packets in the network lost in all interfaces of the TCN routers.

As applied to the routing model presented by the expressions (1)-(5), the lost packets intensity for the u_i node is calculated according to the formula

$$\lambda_{\eta}^{i}=\sum_{j=1}^{R_i}\lambda^{\langle r\rangle}x_{(i,j)}p_{(i,j)}\ ,$$

where all variables refer to the common k -th flow; R_i is the total number of the output interfaces in the routing node u_i. Here and further the k index, identifying the packets belonging to a definite flow, will be omitted to simplify the notation, but implying that all subsequent expressions should be written for every active flow in the network.

Then the service quality support condition by the reliability index of the packets delivery has the form

$$\sum_{j=2}^{\phi}\lambda_{\eta}^{j}\le\lambda^{\langle r\rangle}p_{\langle add\rangle}\,. \tag{27}$$

Under conditions of the problem the components of the vector T_{η} are partially known: the first coordinate $\tau_1^{(\eta)}$ defines the resulting end-to-end mean delay of the packets which should be no more admissible, i.e., $\tau_1^{(\eta)}\le\tau_{\langle add\rangle}$. In this case the rest elements of vector T_{η} are unknown.

Then the expression (23), taking into account (24), can be converted to the form

$$\left\|\begin{array}{c}\lambda_{(\eta)}^{1}\\ ---\\ \Lambda_{\eta-1}\end{array}\right\|=\left\|\begin{array}{ccc}G_{\pi\eta}^{\langle 4,1\rangle} & | & G_{\pi\eta}^{\langle 4,2\rangle}\\ --- & + & ---\\ G_{\pi\eta}^{\langle 4,3\rangle} & | & G_{\pi\eta}^{\langle 4,4\rangle}\end{array}\right\|\cdot\left\|\begin{array}{c}\tau_{1}^{(\eta)}\\ ---\\ T_{\eta-1}\end{array}\right\|, \tag{28}$$

where $\left\| \begin{array}{ccc} G_{\pi\eta}^{\langle 4,1\rangle} & | & G_{\pi\eta}^{\langle 4,2\rangle} \\ --- & + & --- \\ G_{\pi\eta}^{\langle 4,3\rangle} & | & G_{\pi\eta}^{\langle 4,4\rangle} \end{array} \right\| = G_{\pi\eta}^{\langle 4\rangle}$, in this case $G_{\pi\eta}^{\langle 4,1\rangle}$ is the first element of the matrix $G_{\pi\eta}^{\langle 4\rangle}$.

According to the expression (28) the following equalities are true:

$$\lambda_{(\eta)}^{1} = G_{\pi\eta}^{\langle 4,1\rangle}\ \tau_1^{(\eta)} + G_{\pi\eta}^{\langle 4,2\rangle} T_{\eta-1}\,; \tag{29}$$

$$\Lambda_{\eta-1} = G_{\pi\eta}^{\langle 4,3\rangle} \tau_1^{(\eta)} + G_{\pi\eta}^{\langle 4,4\rangle} T_{\eta-1}\,. \tag{30}$$

Having calculated $T_{\eta-1}$ vector from the equation (30)

$$T_{\eta-1} = \left[G_{\pi\eta}^{\langle 4,4\rangle} \right]^{-1} \left(\Lambda_{\eta-1} - G_{\pi\eta}^{\langle 4,3\rangle} \tau_1^{(\eta)} \right) \tag{31}$$

and having substituted it into the equation (29), we receive

$$\lambda_{(\eta)}^{1} = G_{\pi\eta}^{\langle 4,1\rangle}\ \tau_1^{(\eta)} + G_{\pi\eta}^{\langle 4,2\rangle} \left[G_{\pi\eta}^{\langle 4,4\rangle} \right]^{-1} \left(\Lambda_{\eta-1} - G_{\pi\eta}^{\langle 4,3\rangle} \tau_1^{(\eta)} \right). \tag{32}$$

Then, taking into account the conditions (25) and (27), and also $\tau_1^{(\eta)} \le \tau_{\langle add\rangle}$ requirements, we receive the following inequality

$$\begin{aligned} &\lambda^{\langle r\rangle}(1 - p_{\langle add\rangle}) \le \\ &\le G_{\pi\eta}^{\langle 4,2\rangle} \left[G_{\pi\eta}^{\langle 4,4\rangle} \right]^{-1} \Lambda_{\eta-1} + \left(G_{\pi\eta}^{\langle 4,1\rangle} - G_{\pi\eta}^{\langle 4,2\rangle} \left[G_{\pi\eta}^{\langle 4,4\rangle} \right]^{-1} G_{\pi\eta}^{\langle 4,3\rangle} \right)\ \tau_{\langle add\rangle}. \end{aligned} \tag{33}$$

The inequality (33) together with the expression (27) are the required conditions of the service quality assurance by heterogeneous indices – velocity ($\lambda^{\langle r\rangle}$), time ($\tau_{\langle add\rangle}$) and reliability indices ($p_{\langle add\rangle}$). Moreover, the requirement as to the equality of the circuit delays (22) to zero ensures the minimal and equal mean packages delay for all routes being calculated [13,16], this provides minimization of the delay jitter stipulated by the routing multipath strategy realization.

6. ANALYSIS OF THE OFFERED MULTIPATH ROUTING MODEL WITH SERVICE QUALITY ASSURANCE IN TELECOMMUNICATION NETWORK

To demonstrate the advantages of the offered approach the investigation was carried out; in the course of this investigation the problem of a multipath routing with a further estimation of the end-to-end service quality coefficients was solved for different mathematical models and initial data. In the process of the comparative analysis three most efficient models were considered, in the frameworks of these models the multipath routing was formulated as the optimization problem in the linear and nonlinear programming classes.

The first model (model M1) has the system of linear algebraic equations of the TCN state in its basis [16,22] and it is focused on meeting requirements of the Traffic Engineering concept through the balanced use of the channel resources [2]. Minimum of the TCN transmission path maximal coefficient acts as the criterion of the solution optimality [22]:

$$\min \max_i \left\{ \frac{\lambda^i_{(v)}}{\varphi_i} \right\}. \tag{34}$$

The model M2 is also focused on the balanced use of the network resources, but in contrast to the Model1, it realizes this through the uniform use of the buffer resources. Here the minimum of the queue length being maximal over all network interfaces acts as the optimality criterion

$$\min \max_i \{q_i\}, \tag{35}$$

where the calculated expressions for estimation of the mean length of the queue on the i-th network interface q_i should be consistent with the rules for forming the metric tensor $G_v = \left\| g_v^{ij} \right\|$ projection matrix. For example, in the case of the i-th network interface presentation with the system of mass service $SS/M/1/N$, the expression (14) will be used to calculate the value of q_i, and in case of using the model $M/M/1/N$ [6] the following expression will be used

$$q_i = \frac{\rho_i^v}{1-\rho_i^v} - \frac{\left(N_i^v+1\right)\rho_i^{v\left(N_i^v+1\right)}}{1-\rho_i^{v\left(N_i^v+1\right)}}. \tag{36}$$

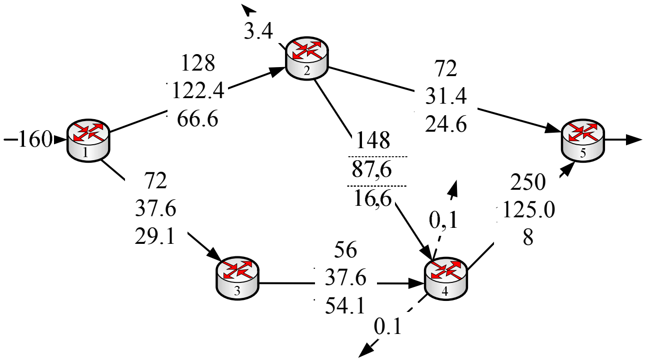
Model1 supplemented by the QoS-conditions (33), obtained in the frameworks of the tensor description of the TCN, was used as the basis for the model M3. Such a set of the optimization problem with the criterion (36) and limitations (1)-(5), (22), (27) and (33) is characteristic for the class of nonlinear programming problems owing to nonlinearity of the majority of conditions, for example, (1) or (33).

The end-to-end multipath delay of the packets D appeared as the main index by which the comparison between different versions of solutions had been carried out. Expression [23] was used as an estimate of the resulting end-to-end multipath delay D, as in the general case in the framework of the multipath routing along each j-th route its own delay d_j will be observed.

$$D = \max\left(d_1, d_2, \ldots, d_j, \ldots, d_R\right), \tag{37}$$

where R is an amount of the routes used for packets transmission.

It should be noted that the mean packages delay d_j along the j-th route was defined as a sum of mean delays in the transmission paths forming the given route. In this case the quantitative estimate of the mean delay in the i-th transmission path was evaluated according to the functional model accepted for description of the corresponding interface: for example, according to expression (10) in case of using the model $M/M/1/N$ and expression (13) in the event that the i-th network interface is presented with the system of the mass service $SS/M/1/N$.

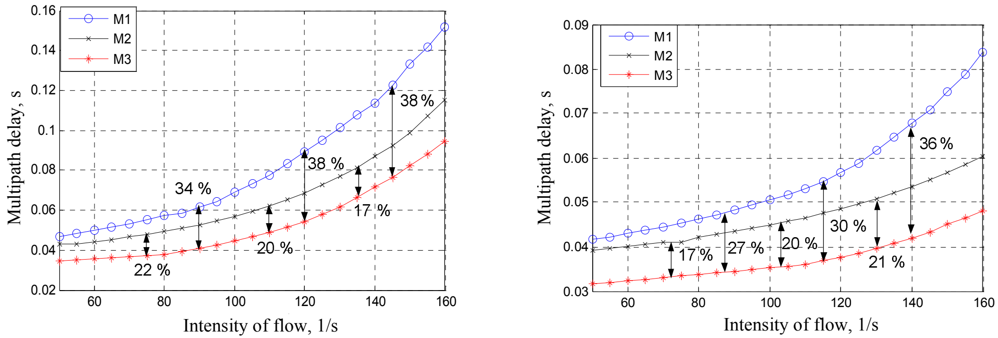


The resulting service quality indicators:
– intensity 156.4 1/s;
– average multi-path delay 91.2 ms;
– probability of losses 0.0225.

FIG. 2: Example of the network and the resulting order of packets routing in it

In the course of investigation with the use of the M1–M3 models the multipath routing problem was solved for a number of TCN varying in structure, amount of nodes, transmission paths and their parameters. Figure 2 demonstrates one of the network structural versions. With the aim of vivid presentation let us restrict our consideration to one-product case when the network serves one flow, for example, from the first node to the fifth one. In this case the QoS-requirements $\lambda^{\langle r\rangle}$, $\tau_{\langle add\rangle}$, $p_{\langle add\rangle}$ act as the initial data and the result of solution is the order of the multipath routing with the flows distribution by separate TCN transmission paths. For example, Fig. 2 demonstrates the problem solution results under the following requirements: $\lambda^{\langle r\rangle}=160$ 1/s, $\tau_{\langle add\rangle}=92$ ms and $p_{\langle add\rangle}=3\cdot 10^{-2}$, and under condition that the buffer volume on each of interfaces of the network routers contains 19 packets ($N=20$). In the breaks of the arks, representing the transmission paths, their throughput (1/s), intensity of the flow in them (1/s) and mean delay of the packets (mc), are shown (from the top down) in the Figure**.** Intensity of the lost packets on the routers interfaces is shown with a dash line.

As a result of the obtained distribution of the traffic in the TCN three paths were calculated: 1-2-5; 1-2-4-5; 1-3-4-5 (in numerical order of the routers). Along each of the paths a mean delay of packets was an identical one and came to 91.2 mc. Having calculated the total losses on the TCN routers (Fig. 2) it is arguable that the probability of packets losses in the network came to $2.25\cdot 10^{-2}$ as a whole. Thus, all requirements on ensuring of the specified numerical values of diverse QoS indicators are met.



a) b)

FIG. 3: Dependence of the end-to-end multipath delay on the intensity of the flow arriving to the network with $p_{\langle add\rangle}=1\cdot 10^{-3}$ under conditions of the model $M/M/1/N$ (a) and $SS/M/1/N$ (b)

Figure 3 demonstrates the dynamics of variation of the end-to-end multipath delay D in the frameworks of the models M1–M3 for the network presented in Fig. 2, depending on intensity of the packets $\lambda^{\langle r\rangle}$ flow arriving to the network with a fixed value of the tolerable losses $p_{\langle add \rangle} = 1 \cdot 10^{-3}$. The plot covers the case of the network loading from 0.25 to 0.8 that satisfies the most common state of the telecommunication network.

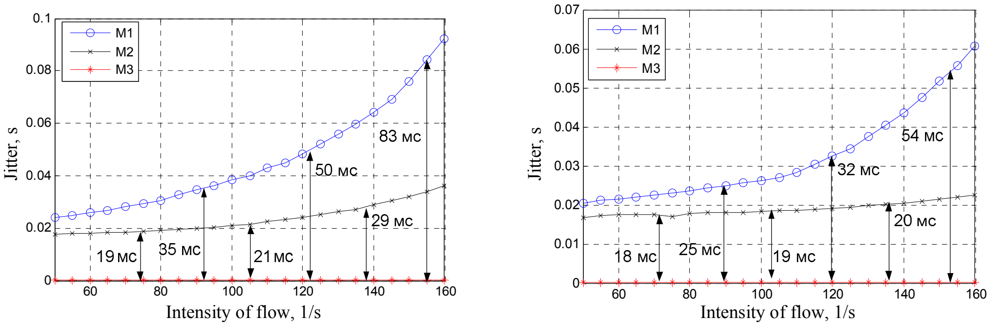
According to the investigations' results the routing solutions, obtained in the frameworks of the M3 model due to the tensor limitations (33) implementation, ensure the gain in the multipath delay D as compared to the M1 and M2 models for the value from 15 to 40% with the same intensity of the flow arriving to the network and the value of losses. In this case the main factor defining the gain value has proved to be the input flow intensity, with its rise the M3 model efficiency increases, indicating to the field of high network loads (0.6–0.8) as a preferable field of this model application. The obtained results make it possible to draw another no less important conclusion: with the same requirements to the quality service in terms of the delay $\tau_{\langle add \rangle}$ and losses probability $p_{\langle add \rangle}$ the model M3 ensures higher speed of transmission (on the average on 40–50% in the area of low and mean loads and at least on 10–20% in the area of high loads).

Peculiarity of the multipath routing consists in the presence of the jitter J (multipath jitter), due to the difference in the delay of packets transmitted along different paths. Jitter J, in the course of simulation by the analogy with [24], was calculated as

$$J = \max_{i,j} \left\{ \left| d_i - d_j - \frac{1}{2} \min\left(\frac{1}{\lambda_{(p)}^{i}}; \frac{1}{\lambda_{(p)}^{j}} \right) \right| \right\},\ i,j = \overline{1,M}. \tag{38}$$

where $\lambda_{(p)}^{i}$ is the intensity of the flow transmitted by the i-th route.

As would be expected the value of the multipath jitter J at routing according to the models M1 and M2 was increasing with the increase in the intensity of the flow arriving to the network, whereas the jitter was lacking for the model M3 (Fig. 4).



a) b)

FIG. 4: Dependence of jitter on intensity of flow arriving to the network with $p_{\langle add \rangle} = 1 \cdot 10^{-3}$ under conditions of the model $M/M/1/N$ (a) and $SS/M/1/N$ (b)

As the routing problem is one of the effective means of ensuring the required quality of service, i.e., the required speed of transmission $\lambda^{\langle r\rangle}$, specified (admissible) end-to-end delay $\tau_{\langle add\rangle}$ and probability of losses $p_{\langle add\rangle}$, then the efficiency of the routing solutions can be estimated using such important indicator as probability of the complex meeting of the whole set of QoS-requirements P_{QoS}. It is in fact the probability that the resulting end-to-end mean delay of the packets will be within one-side confidence interval limited by $\tau_{\langle add\rangle}$, and the packets loss probability at the same time will be within one-side confidence interval limited by $p_{\langle add\rangle}$. In the frameworks of the introduces above designations the probability P_{QoS} was defined according to the expression

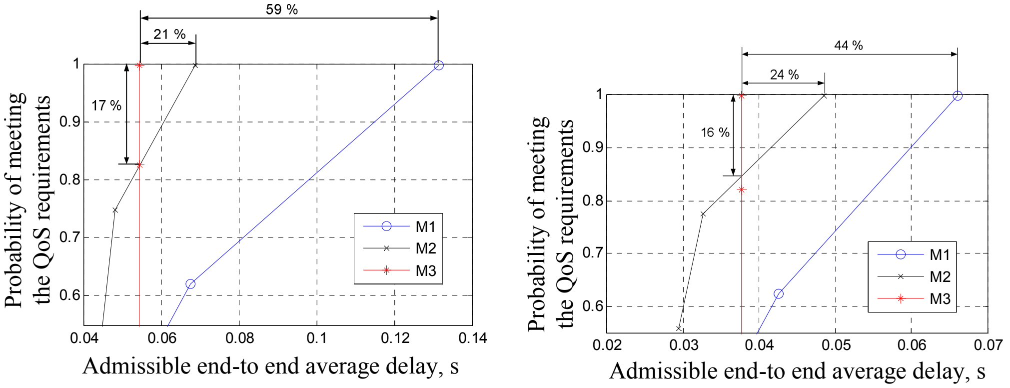
$$P_{QoS} = \sum_{i=1}^{M} Q^i_{(p)}\,, \qquad Q^i_{(p)} = \begin{cases} 0, \text{ if } d_i > \tau_{\langle add\rangle} \text{ and } \mathrm{p}_i > p_{\langle add\rangle}; \\ \dfrac{\lambda^i_{(p)}}{\lambda^{<r>}}, \text{ otherwise,} \end{cases} \tag{39}$$

where $Q^i_{(p)}$ is a share of the flow moving along the i-th route for which the following condition is met

$$d_i \le \tau_{\langle add\rangle} \text{ and } p_i \le p_{\langle add\rangle}. \tag{40}$$

Thus, if along the whole routes the condition (40) is met than the probability of meeting QoS requirements was equal to unity. If along some route the mean delay of packets or the probability of losses exceeded the allowable limit than this resulted in the corresponding decrease in the probability P_{QoS} (39) per a part of the flow being transmitted by this path as along it the specified quality of service was not ensured.

As is shown in Fig. 7, the probability of meeting the QoS requirements is a function of the level of requirements imposed on the QoS, i.e., on the admissible end-to end average delay $\tau_{\langle add\rangle}$ and losses probability $p_{\langle add\rangle}$, as well as on the intensity of the flow arriving to the network. In this case it was found as a result of simulation that with increasing of requirements to the service quality (or with increasing of the load) the probability of meeting QoS requirements P_{QoS} decreases gradually in the frameworks of the modes M1 and M2: gradual exclusion of routes takes place in view of their inability to change the QoS requirements (40).



a) b)

FIG. 7: Dependence of probability to meet QoS requirements up to admissible end-to-end mean delay with $p_{\langle add\rangle} = 1\cdot 10^{-3}$, $\lambda^{\langle r\rangle} = 120$ 1/c under conditions of model $M/M/1/N$ (a) and $SS/M/1/N$ (b)

When using the model M3 the dependence for meeting the QoS requirements has a single-stage form (Fig. 7), which is explained by the following. Model M3 is focused on ensuring equal QoS-indices along the whole set routes (Fig. 6), and the observed value $P_{QoS} = 1$ means the achievement of this goal**.** While the routes exclusions take place with the increase in requirements in the frameworks of the models M1 and M2 (the probability P_{QoS} decreases), the model M3 makes it possible to fulfill requirements completely thereby hold the probability P_{QoS} on the unit level. On the other hand, with very severe requirements to the service quality, the model M3 owing to the limited network resources is not able to ensure equality of the QoS indicators along the whole calculated routes and, as a result, the probability to meet the QoS-requirements is reduced to zero. On the whole the M3 model application makes it possible to increase the probability P_{QoS}, for example, for the structure from Fig. 2 by up to 17% as compared to the M1 model and up to 50% as compared to the model M2 with the same requirements to the service quality, i.e., to increase guarantees of QoS (Fig. 7), thereby to reduce the number of denials of service caused by failure to comply with a specified level of quality.

7. CONCLUSIONS

Multipath routing threading model with the guarantees of the service quality is offered in this work. The model novelty consists, first of all, in the transition to an updated version of conditions of the flow maintaining (1) taking into account the possibility of losing packets caused by the queue buffer overflow in the TCN routers. Secondly, account of such losses has entailed the revision of restrictions associated with the service quality. Owing to tensor formalization of the TCN model it has turned out possible to get in the analytical form the conditions of maintenance of the service quality simultaneously by a set of diverse indicators, namely, transmission speed, mean delay and packets losses probability. It is significant to note that the obtained conditions of the service quality maintenance (33) are invariant ones in its form and do not depend on the used models of the traffic and the packages service on the TCN routers interfaces. The packets service model variation results only in modification of the metrics (12) introduced when sealing the network.

The results of the numerical analysis of the inferred and formerly known solutions from the service quality standpoint have demonstrated the advantages of the tensor approach to the routing problems simulation; this manifested itself in improvement of the end-to-end multipath delay (in average by 15–40%) or in transmission speed rise (by 10–50%)**,** minimization of the of the packet jitter caused by realization of the routing multipath strategy, enhancement of the probability to meet QoS requirements and decrease of the service denials caused by failure to comply with a certain level of quality.

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